MASARYK UNIVERSITY
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Deciding Non–local Choice in High–level Message Sequence Charts

BACHELOR THESIS

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Brno, Spring 2009

Declaration

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Acknowledgement

I would like to thank my advisor RNDr. Vojtěch Řehák, Ph.D. for help and guidance throughout the work on this thesis.

Abstract

Non–local choice property in Message Sequence Chart formalism is causing unwanted and unspecified behavior in the implemented system.

This work focuses on detecting non–local choice in High–level Message Sequence Chart specifications. We propose an algorithm that detects non–local choice. We introduce a new approach to deal with non–local choice, by having further assumptions on the system.

The approach cannot be applied in general, but we can algorithmically decide, whether our approach is appliable. Moreover, when we cannot apply our approach, we may advise the designer, what is causing the inapplicableness.

Keywords

MSC, message sequence chart, MSG, message sequence graph, HMSC, high–level message sequence chart, hierarchical message sequence chart, NLC, non–local choice.

Contents

Introduction

The Message Sequence Chart formalism [6] is a widely used mechanism to design message passing systems, such as network protocols and inter–process communication. Because the formalism provides visual and textual representation, the design may be done in a very intuitive way without loosing the possibility to automatically verify and test the design.

Verification of a complex system in the early stage of development may help to discover potential problems and is therefore in a great interest of designers.

Many properties that can lead to difficulties have been studied, such as *race conditions* [1, 4, 5], *boundedness* [2], *non–local choice* [9], *deadlock*, *livelock*, *cycles*. Some of these properties were proved to be undecidable, but there are also positive results e.g. the *trace race problem* [13].

There are tools developed to perform verification on the designs, such as *uBet* [14], *MSCan* [10] and *SCStudio* [12] that is being developed at Faculty of Informatics, Masaryk University.

This work focuses on the *non–local choice* problem, that appears in a Message Sequence Chart specification, when multiple behaviors are possible, but the decision, which behavior will be executed, has to be performed by distributed processes. One possible solution is to perform an explicit synchronization. Otherwise, this behavior cannot be implemented without unwanted and unspecified behavior of the system.

In this work, we present a new approach dealing with non–local choices that is appliable on certain Message Sequence Chart specifications and is having reasonable demands on the underlying system.

Chapter 1

Foundations

The Message Sequence Charts formalism is a textual and graphical language for describing interaction among parallel components of a distributed system. The formalism was introduced in International Telecommunications Union (ITU–T) Recommendation Z.120 [6]. In this chapter we introduce and define needed terms.

1.1 Basic Message Sequence Charts

Every basic Message Sequence Chart (*MSC*) describes a finite execution of a message–passing system. In a visual *MSC* representation each process of the system is represented by a vertical line. Message exchange is denoted as an arrow from the source process to the destination process. Because we assume asynchronous communication, we need to partition the message exchange into two events – send and receive events. The ordering of the messages from the top of the diagram determines the desired sequence of messages.

There are many various definitions of basic Message Sequence Chart (*MSC*). We will use a definition from [5].

Definition 1. *An* MSC *is defined as a tuple* $(E, <, \mathcal{P}, \mathcal{T}, P, \mathcal{M})$ *where*

- E *is a finite set of* events*;*
- $<$ *is a partial ordering on* E *called* visual order*;*
- $\mathcal{P}$ *is a finite set of* processes*;*
- $\mathcal{T} : E \to \{send, receive\}$ *is a labeling function dividing events into two types* — send *and* receive*;*
- $P : E \to \mathcal{P}$ *is a mapping that associates each event with a process;*
- $\mathcal{M} \subseteq (\mathcal{T}^{-1}(send) \times \mathcal{T}^{-1}(receive))$ *is a bijective mapping, relating every send with a unique receive, such that for any* $(e, f) \in \mathcal{M}$ *we have*

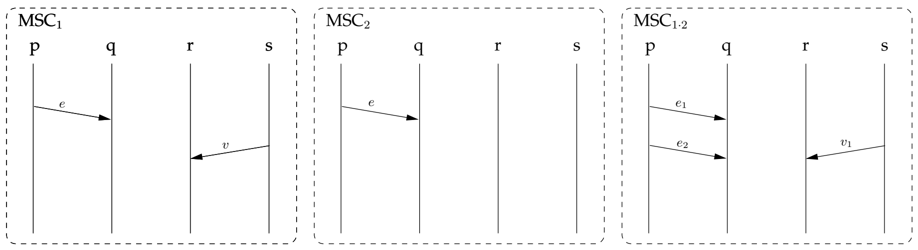


Figure 1.1: MSC concatenation

$P(e) \neq P(f)$ – a process cannot send messages to itself;

Visual order $<$ is defined as the reflexive and transitive closure of

$$\mathcal{M} \cup \bigcup_{p \in \mathcal{P}} <_p$$

where $<_p$ is a total order on $P^{-1}(p)$.

1.2 High–level Message Sequence Charts

To design complex systems the expressivity of *MSCs* may not be sufficient. When designing a complex system, we may need to specify alternations and potentially infinite behavior. That is the reason for introducing High–level Message Sequence Charts (*HMSC*), which allow us to combine *MSCs*. An *HMSC* can be imagined as a directed graph with two types of nodes — *MSCs* and supernodes containing *HMSCs*. The recursion may be only finite, so it is always possible to unfold supernodes and obtain a semantically equivalent *HMSC* with nodes labeled only by *MSCs* as [2] shows. We will refer to *MSC* labeled *HMSCs* as Message Sequence Graphs (*MSG*).

Every *MSG* has to contain two special nodes — an initial node and a terminal node. Every path from the initial node to the terminal node uniquely identifies an *MSC* by sequential composition of *MSCs* on the path. To define everything more formally, we will use a definition from [2].

Definition 2. *An* MSG *is defined as a tuple $G = (\mathcal{S}, \tau, s_0, s_f, L)$ where*

- *$\mathcal{S}$ is a finite set of states.*
- *$\tau \subseteq \mathcal{S} \times \mathcal{S}$ is an edge relation.*

- *$s_0 \in \mathcal{S}$ is an initial state.*
- *$s_f \in \mathcal{S}$ is a terminal state.*
- *$L : \mathcal{S} \to MSC$ is a labeling function;*

A finite sequence of nodes starting with an initial node and ending with a terminal node that has been constructed according to a path in the graph is called a run.

Definition 3. *Given an* MSG $G = (\mathcal{S}, \tau, s_0, s_f, L)$ *a finite sequence of states* $s_1, s_2, \ldots, s_k$*, where* $\forall\, 1 \leq i < k : (s_i, s_{i+1}) \in \tau$*, is a path. A path, where* $s_1 = s_0$ *and* $s_k = s_f$*, is a run.*

We need to show how an *MSG* identifies a set of possible *MSCs*. It will be done by assigning an *MSC* to every run. This is achieved by sequential composition of *MSCs* on the path. It is sufficient to show how to compose two *MSCs*.

Definition 4. *Given two* MSCs $M_1 = (E_1, <_1, \mathcal{P}, \mathcal{T}_1, P_1, \mathcal{M}_1)$ *and* $M_2 = (E_2, <_2, \mathcal{P}, \mathcal{T}_2, P_2, \mathcal{M}_2)$*, let their sequential composition be an* MSC $M_1 \cdot M_2 = ((E_1, 1) \cup (E_2, 2), <, \mathcal{P}, \mathcal{T}_1 \cup \mathcal{T}_2, P_1 \cup P_2, \mathcal{M}_1 \cup \mathcal{M}_2)$*, where* $<$ *is a transitive closure of* $<_1 \cup <_2 \cup \bigcup_{p \in \mathcal{P}} (P_1^{-1}(p) \times P_2^{-1}(p))$*.*

Because we do not synchronize processes when composing, it is possible that some events from the latter *MSC* may be executed before the events from the first *MSC*. See Figure 1.1, sending event of message e_2 may be executed even before the receive event of message v_1.

Definition 5. *Let* $G = (\mathcal{S}, \tau, s_0, s_f, L)$ *be an* MSG *and* $\sigma = s_1, s_2, \ldots, s_k$ *be a path in G. Then* MSC $M_\sigma = L(s_1) \cdot L(s_2) \cdot \ldots \cdot L(s_k)$*.*

The set of all *MSCs* specified by an *MSG* is defined as follows:

Definition 6. *Given an* MSG $G = (\mathcal{S}, \tau, s_0, s_f, L)$*, let* $L(G)$ *be a set of* MSCs*:*

$$L(G) = \{\, M_\sigma \mid \sigma \text{ is a run in } G \}\,.$$

Chapter 2

Non–local Choice

Sometimes we may consider an *MSG* specification to be more than a sequence diagram. We may try to find patterns of constructions that are hard to implement, or can lead to potential errors when implementing.

Detecting such problematic scenarios in an early stage of development prevents the final product of containing errors and ensures smooth implementing of desired behavior.

Non–local choice is a problematic property, occurring when processes from set $\mathcal{P}$ do not share a common memory. Whenever there is a branching node in an *MSG* specification, we want exactly one branch to continue with executing the run. That may be hard to achieve when there are two different branches initiated by different processes from $\mathcal{P}$. These processes should somehow synchronize and agree on a branch that will be executed, but that is not possible without any extra communication, because they do not share any memory. The branching node is called a non–local choice node.

Running an *MSG* with a non–local choice leads to an unwanted and unspecified behavior. We call this behavior an implied behavior. It appears as a consequence of a non–local choice node, when all the processes in different branches start executing their branches. An implied behavior for a non–local choice node can be seen in Figure 2.1.

There have already been approaches how to detect and deal with non–local choice nodes. Unfortunately there have been also many different definitions of non–local choice.

There is an algorithm in [3] detecting non–local choice nodes in *MSG* specification that runs in time linear to the total number of messages exchanged in the *MSG* specification. The speed of algorithm is achieved by further assumption on every *MSC* in *MSG*. However the result looses generality, because it is assumed that every process in every *MSC* exchanges at least one message.

An algorithm to detect non–local choice nodes in a general *MSG* is presented in [11]. The authors aim is to detect implied scenarios, by constructing *LTS* and analyzing them. Unfortunately the authors were unable to

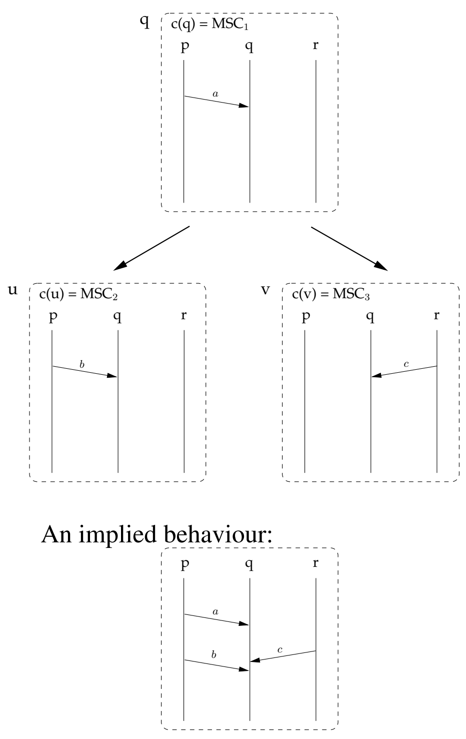


Figure 2.1: Non–local choice node with an implied behavior

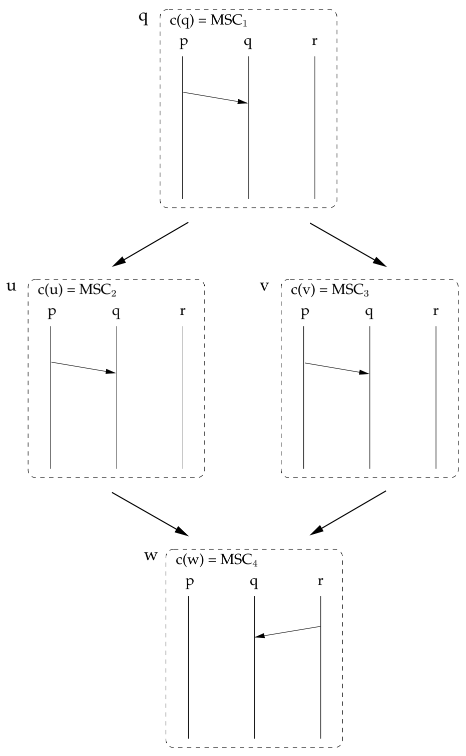


Figure 2.2: Non–local choice

prove the correctness of the algorithm.

A way how to deal with detected non–local choice nodes is introduced in [8]. A fixed arbiter process from $\mathcal{P}$ is chosen. Unspecified implied behavior is allowed in every branch, but it is guaranteed that there exists a position in every branch, where arbiter process decides which branch was chosen and the rest of the communication from other branches is forgotten. This approach is not general — there are *MSGs*, where no such arbiter can be chosen. No algorithm detecting *MSGs*, where this approach can be applied, is presented.

There are more definitions of non–local choice, but we consider the one most describing what the problem is in [9].

To formally define non–local choice we need to introduce some new notions. We will denote by $\text{Min}_B(M)$ a subset of $\mathcal{P}$ that contains all processes that are possessing a minimal event in an *MSC* M.

Definition 7. *Let* $M = (E, <, \mathcal{P}, \mathcal{T}, P, \mathcal{M})$ *be an* MSC. *Then let*

$$Min_B(M) = \left\{ p \in \mathcal{P} \mid (\exists e \in E), (\forall e' \in E) : \neg(e' \leq e)) \wedge P(e) = p \right\}$$

The previous notion can be extended on *MSGs* as follows:

Definition 8. *Let* $G = (\mathcal{S}, \tau, s_0, s_f, L)$ *be an* MSG. *Then let*

$$Min_H(G) = \{ p \in \mathcal{P} \mid (\exists MSC\ H)(H \in Lin(G) \wedge p \in Min_B(H)) \}$$

Given an *MSG* $G = (\mathcal{S}, \tau, s_0, s_f, L)$ and $u \in \mathcal{S}$, let the successor set be $\text{Succ}(u) = \bigcup_{(u,v)\in\tau}\{v\}$.

Finally we are ready to formally define non–local choice.

Definition 9. *Given an* MSG $G = (\mathcal{S}, \tau, s_0, s_f, L)$ *a state* $q \in \mathcal{S}$ *is called a non–local choice node iff* $|Succ(q)| \geq 2$ *and one of the following holds:*

- $\exists\, v_k, v_l \in Succ(q)$*, such that* $Min_H(G_k) \neq Min_H(G_l)$
- $\exists\, v_k \in Succ(q)$*, such that* $|Min_H(G_k)| \geq 2$.

where:

$$MSG\ G_k = (\mathcal{S}, \tau, v_k, s_f, L)$$
$$MSG\ G_l = (\mathcal{S}, \tau, v_l, s_f, L)$$

2.1 Algorithm to detect Non–local choice nodes in HMSC

As no suitable algorithm with correctness proof on general *MSGs* is known, we present our algorithm that detects non–local choice nodes.

The idea behind the algorithm is to compute for every node in *MSG* a subset of processes that initiate the behavior directly after this node. As we want this algorithm to work on general *MSGs*, we do not assume that every process in every *MSC* exchanges a message. Therefore looking only at the successor set may not be enough. Whenever there is an idle process (does not exchange a message) in an *MSC*, it is possible that the initiative in the communication is propagated from some *MSC* deeper in the graph.

This leads to our iterative Algorithm 1, that improves this set until it reaches a fixed–point. When we have this set for every node it is easy to detect non–local choice nodes.

2.1.1 Total correctness

In the following section will be shown, that the algorithm is totally correct. We need to show that the algorithm terminates and is partially correct.

Algorithm 1 Detect Non–local Choice nodes

```
1: {Initialization}
2: for all $q \in \mathcal{S}$ do
3:     $q_0$.MEP $\leftarrow$ Min$_B(c(q))$
4:     $q$.IP $\leftarrow$ Idle processes in $c(q)$
5: $i \leftarrow 0$
6: {Iteration}
7: repeat
8:     $i \leftarrow i + 1$
9:     for all $q \in \mathcal{S}$ do
10:        $q_i$.MEP $\leftarrow ((\bigcup_{v \in \text{Succ}(q)} v_{i-1}.\text{MEP}) \cap q.\text{IP}) \cup q_{i-1}.\text{MEP}$
11: until $\exists q \in \mathcal{S} : (q_i.\text{MEP} \neq q_{i-1}.\text{MEP})$
12: {Detect NLC nodes}
13: for all $q \in \mathcal{S}$ do
14:     $q$.NLC $\leftarrow$ FALSE
15:     if $(|\text{Succ}(q)| \geq 2)$ then
16:         if $(\exists v \in \text{Succ}(q) : |v_i.\text{MEP}| \geq 2)$ then
17:             $q$.NLC $\leftarrow$ TRUE
18:         else if $(\exists v, w \in \text{Succ}(q) : v_i.\text{MEP} \cap w_i.\text{MEP} = \emptyset)$ then
19:             $q$.NLC $\leftarrow$ TRUE
```

Termination The only part of the code that can prevent the algorithm from terminating is the iteration phase (lines 7–11). We assign to each iteration its state, which is defined as the value of variable q.MEP for all $q \in \mathcal{S}$ at line 11. More formally a state of the i–th iteration is defined:

$$Q_i = \prod_{q \in \mathcal{S}} q_i.\text{MEP}$$

There is only a finite number of different states. So it remains to show, that the algorithm does not cycle on these states.

Lemma 1. *Let $0 \leq i \leq j$, then the following holds:*

$$\forall q \in \mathcal{S} : q_i.MEP \subseteq q_j.MEP$$

Proof. Follows directly from the variable computation. □

Suppose there is a sequence of indexes $u \leq v \leq w$ during the run, such that $Q_u \neq Q_v$ and $Q_u = Q_w$. Then there exists a $q \in \mathcal{S}$, such that:

$$(q_u.\text{MEP} \subset q_v.\text{MEP})$$

It follows from Lemma 1 that during the computation, we cannot reach state Q_w. So we reached a contradiction with the assumption. Therefore the algorithm will terminate, when some state is repeated for the first time.

Partial correctness Suppose that *MSG* $G = (\mathcal{S}, \tau, s_0, s_f, L)$ is the input of the algorithm. During the computation the following invariant holds:

$$\forall q \in \mathcal{S}, \forall p \in \mathcal{P}, \forall i \geq 0 : p \in q_i.\text{MEP} \Rightarrow p \in \text{MIN}_H(G_q).$$

Moreover, when $Q_i = Q_{i+1}$, then

$$\forall q \in \mathcal{S}, \forall p \in \mathcal{P} : p \in q_i.\text{MEP} \Leftrightarrow p \in \text{MIN}_H(G_q),$$

where *MSG* $G_q = (\mathcal{S}, \tau, q, s_f, L)$.

Proof. We will prove the first statement by induction with respect to the number of iterations. Let $p \in \mathcal{P}$ and $q \in \mathcal{S}$ be arbitrary, but fixed.

Base case $p \in q_0.\text{MEP} \Rightarrow p \in \text{Min}_B(L(q)) \subseteq \text{Min}_H(G_q)$

Induction step $p \in q_{i+1}.\text{MEP}$

There are two possibilities to consider:

$p \in q_i.\text{MEP} \;\Rightarrow p \in \text{Min}_H(G_q)$ (induction hypothesis)

$$\begin{aligned} p \notin q_i.\text{MEP} \;\Rightarrow\;\; & p \in \textstyle\bigcup_{v \in \text{Succ}(q)} (v_i.\text{MEP} \cap q.\text{IP}) \\ \Rightarrow\;\; & \exists v \in \text{Succ}(q) : p \in v_i.\text{MEP} \cap q.\text{IP} \\ \Rightarrow\;\; & \text{(induction hypothesis)}\, p \in \text{Min}_H(G_v) \wedge p \in q.\text{IP} \\ \Rightarrow\;\; & p \in \text{Min}_H(G_q) \end{aligned}$$

where *MSG* $G_v = (\mathcal{S}, \tau, v, s_f, L)$, *MSG* $G_q = (\mathcal{S}, \tau, q, s_f, L)$.

Now we will prove the second statement. The $\Rightarrow$ case follows from the first statement. The $\Leftarrow$ case will be proved by contradiction.

We will fix the i–th iteration and denote by q.MEP the value of q_i.MEP. Assume $p \in \text{Min}_H(G_q) \wedge p \notin q.\text{MEP}$. Because $p \in \text{Min}_H(G_q)$ there exists a path $s_0, \ldots, s_n$, such that $s_0 = q, p \in s_n.\text{MEP}$ and for $0 \leq j \leq (n-1)$: $p \notin s_j.\text{MEP}$ and $p \in s_j.\text{IP}$. Therefore the algorithm would add process p to the set $s_{(n-1)}$.MEP in the next iteration. But that is not possible, because $Q_i = Q_{i+1}$. So there is no such node and the length of the path is one. But that is a contradiction with the assumption, because $p \in s_n.\text{MEP} = s_1.\text{MEP} = q.\text{MEP}$. □

Chapter 3

Decidable Non–local Choice

In this chapter we will present a new approach to deal with non–local choice nodes. We will assume that we are able to add some additional bounded information to every message. And that each process is able to store some bounded information.

The non–local choice will be resolved by adding additional data into preceding communication. These data will be distributed among all the processes, that are involved in initiating the communication directly after the non–local choice node.

This work describes resolving non–local choice only for one arbitrary but fixed non–local choice node q. To resolve the whole *MSG* specification the procedure has to be performed for every non–local choice node.

We can assign to the non–local choice node q in *MSG* $G = (\mathcal{S}, \tau, s_0, s_f, L)$ a set of processes, to which we need to distribute the information. We will call this set *Trigger*.

$$\text{Trigger}(q) = \bigcup_{v \in \text{Succ}(q)} \text{Min}_H(G_v)$$

where *MSG* $G_v = (\mathcal{S}, \tau, v, s_f, L)$.

We will say, that a send event e in *MSC* $L(w)$ can resolve a non–local choice node q in *MSG* G, if there exists a run going through w and q, such that the information added to message $(e, f) \in \mathcal{M}$, can be distributed during the run to all processes from *Trigger*(q), by adding the information to existing communication.

Definition 10. *Let $G = (\mathcal{S}, \tau, s_0, s_f, L)$ be an* MSG, *q a non–local choice node, $\sigma = s_0, \ldots, w, \ldots, q, \ldots, s_f$ a run in G, an event e from* MSC *$L(w)$ is a resolving event for q on σ, iff $\mathcal{T}(e) = send$ and* $\text{MSC}_{w\ldots q} = (E, <, \mathcal{P}, \mathcal{T}, P, \mathcal{M})$ *has the following property:*

$$\forall p \in \textit{Trigger}(q), \exists e_p \in P^{-1}(p) : e < e_p$$

The foregoing definition covers events that are able to distribute additional information among processes in set *Trigger*(q). We would like to detect events that resolve non–local choice as late as possible. We will name these events deciding events.

Definition 11. *A resolving event* e *for* q *on* σ *is a deciding event iff for all other resolving events* e' *for* q *on* σ *condition* $\neg(e < e')$ *holds.*

Proposition 1. *Let* σ *be a path without a deciding event for* Trigger(q) *and* σ' *a subsequence of* σ*. Then* σ' *does not possess a deciding event for* Trigger(q).

Let σ' *be a path with a deciding event for* Trigger(q) *and let* σ' *be a subsequence of* σ*. Then* σ *possesses a deciding event for* Trigger(q).

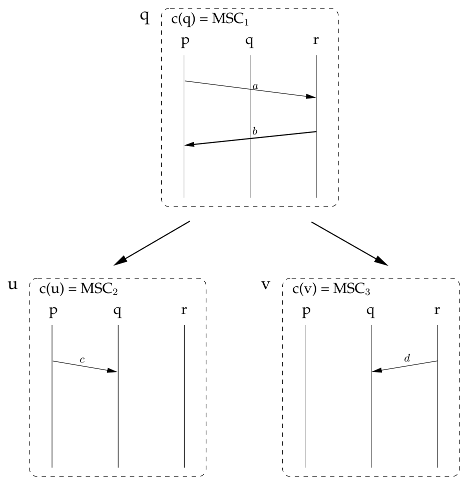


Figure 3.1: Deciding NLC node q, in event b

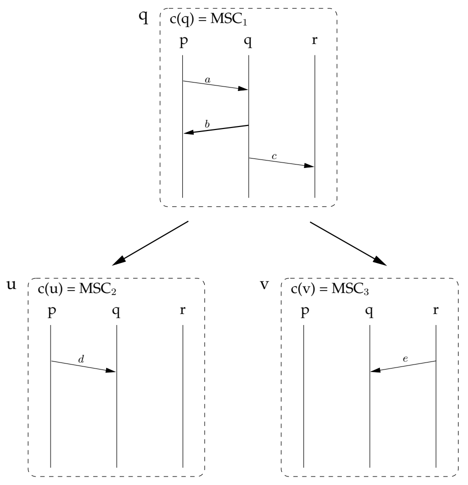


Figure 3.2: Deciding NLC node q, in event b

Suppose we will find deciding events on every possible path from the initial node. That would basically mean that it doesn't matter how we reached a non–local choice node, because we would always be able to add additional information, which solves the non–local choice problem. Clearly not all possible *MSGs* are having the needed property. See Figure 2.1.

But even when all paths from the initial node to a non–local choice node are possessing a deciding event, we may get into trouble when implementing such behavior.

Consider Figure 3.3. Every path incoming to a non–local choice node q contains node p with a deciding event a for q. The amount of information that needs to be added to message a may rise above any given bound, because a single occurrence of the deciding message a needs to decide potentially infinite occurrences of the non–local choice node q.

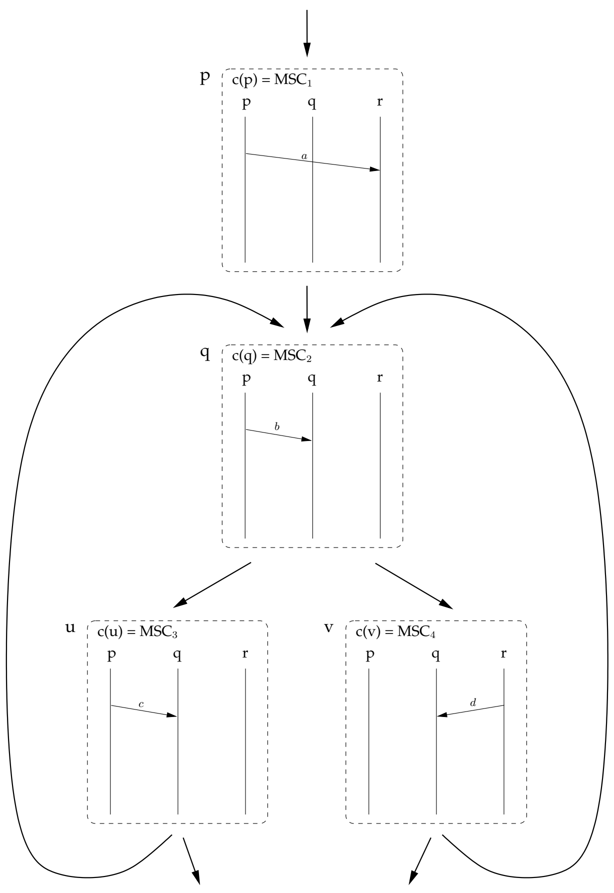


Figure 3.3: Problematic non–local choice

The previous requirements lead to the following definition.

Definition 12. *A non–local choice node q is called decidable iff every path from initial node to node q possesses a deciding event and there exists a constant k for node q, such that every occurrence of a deciding event has to decide less or equal than k occurrences of the non–local choice node q.*

We will refer to property that additional information is bounded as to boundedness of the deciding event.

We will show how to check whether a given *MSG* G with a non–local choice node q is decidable. We will have to check two properties:

1. Every path from initial node to node q possesses a deciding event.
2. Every occurrence of a deciding event has to decide less or equal than k occurrences of the non–local choice node.

3.1 Checking paths for deciding events

Definition 13. *Let q be a non–local choice node in* MSG G*, then let*

$$\mathit{Path}(G, q) = \{\sigma \mid \sigma \ \text{ is a path from initial node to } q\}$$

We need to check that every *MSC* M_σ for every $\sigma \in \mathrm{Path}(G, q)$ contains a deciding message for q. But all possible elements of $\mathrm{Path}(G, q)$ cannot be generated, because this set may be potentially infinite.

Definition 14. *A path σ in the directed graph $G = (V, E)$ is called acyclic, if each vertex $v \in V$ has at most one occurrence in σ.*

The following lemma shows that it suffices to check only a finite subset of all possible paths.

Lemma 2. *There exists an* MSC M_σ *for $\sigma \in \mathit{Path}(G, q)$ without a deciding event iff there exists an* MSC $M_{\sigma'}$ *without a deciding event for acyclic $\sigma' \in \mathit{Path}(G, q)$.*

Proof.

$\Leftarrow$ Obvious.

$\Rightarrow$ We will show converse. Let all *MSC* $M_{\sigma'}$ with acyclic $\sigma' \in \mathrm{Path}(G, q)$ contain a deciding event, then all *MSC* M_σ with $\sigma \in \mathrm{Path}(G, q)$ are possessing a deciding event.

Consider a directed graph $G = (V, E)$, where:

- $V = \{v \mid v \text{ is a state from } \sigma\}$
- $E = \{(u, v) \mid u, v \text{ are nodes from } \sigma \text{ and } uv \text{ is a substring of } \sigma\}$

As σ is a path from the initial node to the non–local choice node q, there must exist a shortest path σ' in G from the initial node to q. This path is acyclic, therefore it possesses a deciding event. The path σ' is a subsequence of path σ and by Proposition 1 path σ possesses a deciding event.

□

To check the needed property it suffices to check all paths from the following set.

$$\text{Acyclic–Path}(G, q) = \{\sigma' \mid \sigma' \text{ is an acyclic path from initial node to } q\}$$

The upper bound for the set cardinality will be given in Chapter 4.

3.2 Checking boundedness for deciding events

The requirement to keep the data bounded is quite natural. Otherwise we would force the execution of the *MSG* to loop only for a limited number of iterations, or to leave the non–local choice after bounded decided iterations unsolved.

Whenever an *MSG* contains a loop with a non–local choice node on it, the execution may repeat this loop potentially infinitely many times. So the loop itself has to contain a deciding event for the non–local choice node. But as we will see later not necessarily in the first iteration.

All cycles containing a fixed non–local choice node q have to be checked. Therefore we may restrict the whole *MSG* only to the strongly connected component containing q.

Definition 15. *A directed $G = (V, E)$ is called strongly connected iff for arbitrary two vertices $u, v \in V$, there exists a path from u to v and from v to u. Strongly connected component containing vertex q is a maximal strongly connected subgraph containing q.*

As the set of all cycles from some non–trivial strongly connected component is always infinite, we will use a variant of Lemma 2 and will check only distinct elementary cycles.

Definition 16. *A cycle is elementary if no vertex appears twice. We will call two cycles distinct if one is not a cyclic permutation of the other.*

However the situation may get more difficult. One might think that when there is an elementary cycle without a deciding event it is sufficient to have an unbounded deciding event.

This situation can be seen in Figure 3.4. Node q is a non–local choice node and qv is an elementary cycle without a deciding event, but cycle $qvqv$ possesses a deciding event (Figure 3.5). And it is clear that if we will iterate the cycle, we will generate new deciding events for the non–local choice node q and each of these events will be bounded.

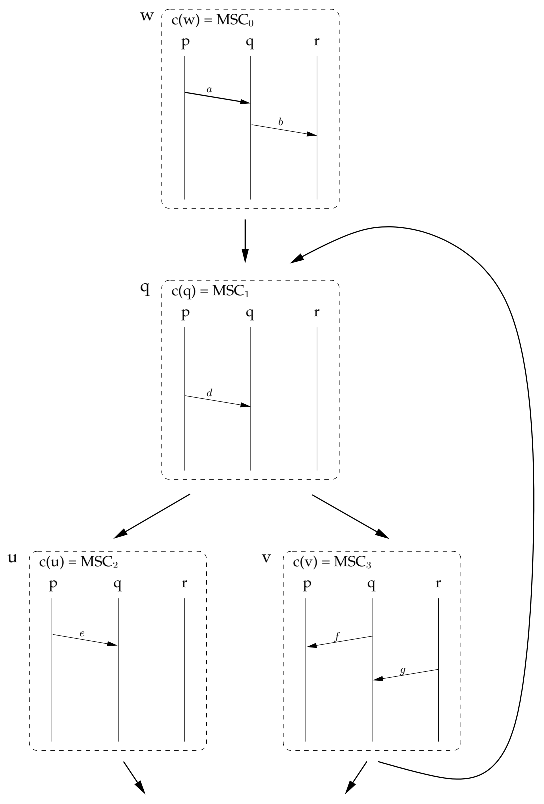


Figure 3.4: More iterations are needed

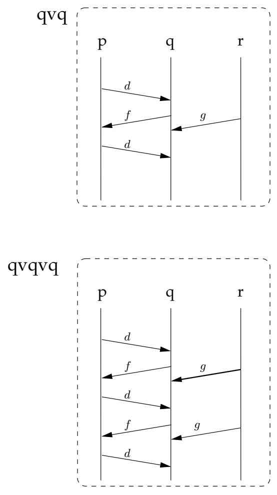


Figure 3.5: Generating deciding events

We may leave the decision of the first occurrences of non–local choices on the deciding events on incoming acyclic paths, and when the cycles begin to generate their own deciding events, it will be possible to decide on the cycles.

It remains to characterize *MSGs* containing unbounded deciding events. And decide whether a cycle willl generate deciding events.

3.2.1 Characterizing *MSGs* containing unbounded deciding events

As the property of having an unbounded deciding event is quite difficult to handle, we will prove the following theorem that characterizes those *MSGs* that contain an unbounded deciding event. We will assume from the previous part, that the *MSG* is already checked and it contains a deciding event on every path from the initial node to the non–local choice node.

Theorem 1. *Given an* MSG $G = (\mathcal{S}, \tau, s_0, s_f, L)$*, with a deciding event on every path from* s_0 *to the non–local choice node* q*, then there exists an unbounded deciding event iff there exists an elementary cycle* c *in graph* $G' = (\mathcal{S}, \tau)$ *containing node* q *such that* $\forall n \geq 0$*,* MSC M_{c^n} *doesn't possess a deciding event.*

Proof.

$\Leftarrow$ As node q is reachable, there must exist a path σ from initial node to node q, with a deciding event e. There also exists an elementary cycle c containing node q such that arbitrary iteration of the cycle c does not contain a deciding event. It means that deciding event e has to decide all occurrences of node q and the number of occurrences may be arbitrary large.

$\Rightarrow$ We have an unbounded deciding event e for non–local choice node q. Let M denote the number of elementary cycles containing node q. And let $k \geq 0$ be an arbitrary but fixed number. Because e is unbounded, there exists a path σ in graph $G' = (\mathcal{S}, \tau)$:

$$\sigma = u\, q\, c_1\, q\, c_2 \ldots q\, c_{M \cdot k+1}\, q$$

We require subpath u to contain all deciding events for node q. We denote by c_i (for all i) an arbitrary cycle starting in q and ending in q, but there is no occurrence of q in c_i. That means we have $M \cdot k + 1$ occurrences of node q in subpath $c_1\, q\, c_2 \ldots q\, c_{M \cdot k+1}\, q$. And there is no deciding event for q in this sequence.

We may replace every cycle c_i by its accordant elementary cycle c'_i. By Proposition 1 the new path $c'_1\, q\, c'_2 \ldots q\, c'_{M \cdot k+1}\, q$ has no deciding event for q.

There exists at least one elementary cycle c, which has at least $k + 1$ occurrences in $c'_1\, q\, c'_2 \ldots q\, c'_{M \cdot k+1}\, q$. Using again Proposition 1, we may strip all other elementary cycles and obtain a path, with $k + 1$ elementary cycles c, without a deciding event. Therefore $\forall n \geq 0$, *MSC* M_{c^n} doesn't possess a deciding event.

□

The previous theorem simplifies detecting of an unbounded deciding event. We need to generate all elementary cycles for a given graph, and check whether they will generate a deciding event. If this is the case we do not have an unbounded deciding event.

3.2.2 Recognizing self-deciding cycles

We will show how to recognize that an elementary cycle will generate deciding events. It is sufficient to focus only on the communication type and the order of the messages is not important. Because by repeating the elementary cycle we can always achieve desired ordering. To handle communication type we introduce the following definition:

Definition 17. *A communication graph for* MSC $M = (E, <, \mathcal{P}, \mathcal{T}, P, \mathcal{M})$ *is a directed graph* $G_M = (\mathcal{P}, E_M)$*, where* $E_M = \{(p, q) \mid \exists e_1, e_2 \in E \ : (e_1, e_2) \in \mathcal{M} \wedge P(e_1) = p \wedge P(e_2) = q\}$

An example of a communication graph of *MSC* M_{qvqvq} from Figure 3.5 can be seen on Figure 3.6

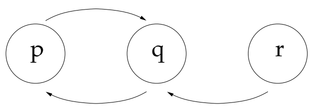


Figure 3.6: Communication graph

We do not need to iterate the original *MSC* because we can decide the property from a communication graph as Lemma 3 shows.

Lemma 3. *Let* c *be an elementary cycle containing non–local choice node* q*, then some iteration of the cycle contains deciding event iff there exists a subgraph of the communication graph* G_c *satisfying the following properties:*

- G_c *is a tree and all the edges are oriented away from the root*
- *all the processes from set* $Trigger(q)$ *are in the tree*

Proof.

$\Rightarrow$ Suppose there exists a number $n \geq 1$ such that *MSC* M_{c^n} contains a deciding event e on a process p. Then for every process $r \in Trigger(q)$, there exists a sequence of messages that distributes the information from p to r. There is an acyclic subgraph of the communication graph containing all processes from $Trigger(q)$. Spanning tree of the graph rooted in p is a subtree with the needed properties.

$\Leftarrow$ There is a sequence of messages from some root process p to every process $r \in Trigger(q)$. We may use these messages to distribute information from p to r. All events on process p are resolving events. Let n be the height of the tree, then we need at most n iterations of the cycle.

□

We have shown how to effectively check whether a given *MSG* G with a non–local choice node q contains an unbounded deciding event. The only remaining thing that needs to be shown is that there exists a bound k not only for elementary cycles but for arbitrary cycles.

Let $\mathcal{C}$ denote all elementary cycles in the strongly connected component of the *MSG* G with the non–local choice node q. And let $Height(c)$ denote the minimal height of all subtrees (satisfying properties stated in Lemma 3) of the communication graph for elementary cycle c. Then we need at most

$$\sum_{c \in \mathcal{C}} Height(c)$$

occurrences of node q, before a deciding event is created. As for all elementary cycles $c : \ Height(c) < |\mathcal{P}|$, we may generalize the upper bound k to:

$$|\mathcal{C}| \cdot |\mathcal{P}|$$

This chapter has shown that we are able to decide some non–local choice nodes by adding additional information into preceding communication. We wanted every deciding event to decide only finite occurrences of the non–local choice node, so that we may easily implement this feature. We have shown that we can algorithmically decide all the needed properties. In Chapter 4, an algorithm and some algorithmic aspects of the problem will be given.

Chapter 4

Algorithms

4.1 Listing elementary cycles

In our approach we need to enumerate all elementary cycles in a given directed graph. We will use an algorithm from [7]. We can slightly modify the algorithm, because we do not need all elementary cycles, but only cycles containing the non–local choice node q.

The complexity of Algorithm 2 lies in $\mathcal{O}((n+e)\cdot(c+1))$. Where n is the number of vertices, e number of edges and c number of elementary cycles.

There may be at most:

$$\sum_{i=1}^{n-1}\binom{n}{n-i+1}(n-i)!$$

elementary cycles on n vertices.

The graph $G=(V,E)$ is represented by an adjacency list $A_G(v)$ for all $v\in V$. To ensure no vertex is used more than once boolean array $blocked(v)$ is used.

4.1.1 Generating all acyclic paths

We may use this algorithm also to generate all acyclic paths from initial node to non–local choice node q in *MSG* $G=(\mathcal{S},\tau,s_0,e_f,L)$. We construct a different graph $G'=(V',E')$, where

- $V'=\mathcal{S}\cup\{v\}$, such that $v\notin\mathcal{S}$
- $E'=\tau\cup\{(q,v),(v,s_0)\}$.

We enumerate all elementary cycles of graph G containing the newly added vertex v. Every elementary cycle of G has to contain vertices s_0, q and uniquely identifies an acyclic path from s_0 to q. We can obtain this path by removing vertex v from the cycle. It is easy to see, that every acyclic path can be constructed in this way.

Algorithm 2 Circuit finding algorithm

```
1: **Global variables**: $stack, blocked, s, B$
2: $stack \leftarrow \emptyset$
3: $s \leftarrow$ non–local choice node $q$
4: for all $i \in V$ do
5:    $blocked(i) \leftarrow FALSE$
6:    $B(i) \leftarrow \emptyset$
7: $CIRCUIT(s)$
8:
9: procedure $CIRCUIT(v)$
10:    $f \leftarrow FALSE$
11:    $stack \leftarrow push(v)$
12:    $blocked(v) \leftarrow TRUE$
13:    for all $w \in A_k(v)$ do
14:       if $w = s$ then
15:          output circuit composed of stack followed by s
16:          $f \leftarrow TRUE$
17:       else if $\neg blocked(w)$ then
18:          if $CIRCUIT(w)$ then
19:             $f \leftarrow TRUE$
20:    if $f$ then
21:       $UNBLOCK(v)$
22:    else
23:       for all $w \in A_k(v)$ do
24:          if $v \notin B(w)$ then
25:             $B(w) \leftarrow B(w) \cup \{v\}$
26:    $stack \leftarrow pop(v)$
27:    return $f$
28: end procedure $CIRCUIT$
29:
30: procedure $UNBLOCK(u)$
31:    $blocked(u) \leftarrow FALSE$
32:    for all $w \in B(u)$ do
33:       $B(u) \leftarrow B(u) - \{w\}$
34:       if $blocked(w)$ then
35:          $UNBLOCK(w)$
36: end procedure $UNBLOCK(u)$
```

4.2 *MSC* data structure

4.2.1 Concatenating *MSCs*

Most of our operations during the algorithm are concatenating *MSCs* along some path in *MSG*. Therefore we want these operations to be fast. We propose a data structure that allows us to concatenate two *MSCs* in time $\mathcal{O}(|\mathcal{P}|)$.

Every event set E in an *MSC* $M = (E, <, \mathcal{P}, \mathcal{T}, P, \mathcal{M})$ with ordering $<$ is a directed acyclic graph. Moreover every vertex in the graph has out–degree less or equal than 2.

Therefore we may store events from E in a double linked list and order the events according to some topological ordering.

Definition 18. *Topological ordering of a partial ordered set* $(E, \prec)$ *is a linear order* $(E, <)$ *such that:*

$$\forall e_1, e_2 \in E : \; e_1 \prec e_2 \Rightarrow e_1 < e_2$$

We need to store the outgoing edges of the vertex. We will do so by assigning two more integers to every vertex. We will call them out_0, out_1. The value denotes the relative position of the edge target in the double linked list. If there are less than two edges we will use a $\perp$ value.

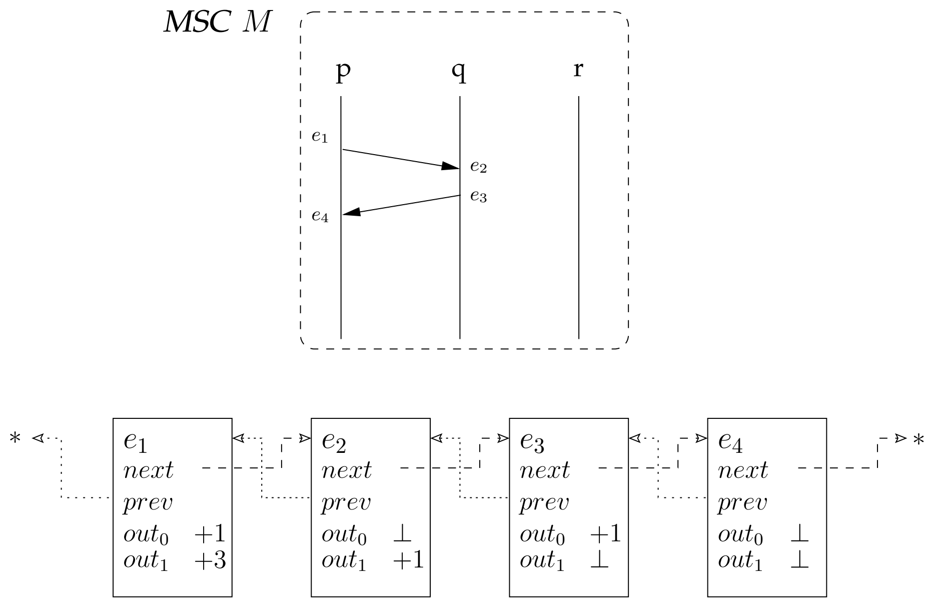
When concatenating two *MSCs* we need to add some order between the maximal events from the earlier *MSC* and the minimal events from the latter *MSC*. Because finding such event may take linear time with respect to the number of events, additional structure in every *MSC* is used.

For *MSC* $M = (E, <, \mathcal{P}, \mathcal{T}, P, \mathcal{M})$, let $Min(M)$ be an array of positions of minimal events in the list. We will also add pointers to these events, to be able to directly access them. Let the array be sorted according to some fixed ordering of $\mathcal{P}$. $Max(M)$ denotes an array of positions of maximal events with pointers to them. If there are no minimal or maximal events on the process in M we will use $\emptyset$ for the position of the event. The last thing that needs to be stored is the length of the list (cardinality of the event set E) in $length(M)$. An example can be seen on Figure 4.1

Given two *MSCs* M_1 and M_2 in the described structure, it will be shown how to compute *MSC* $M_{1 \cdot 2}$ in time $\mathcal{O}(|\mathcal{P}|)$.

1. Append M_2 list at M_1 (in $\mathcal{O}(1)$).

2. $\forall p \in \mathcal{P}$ add edge between the maximal event on p in M_1 and the minimal in M_2, if there is some. Because we have pointers to these events and know the length of the lists, this takes only $\mathcal{O}(|\mathcal{P}|)$ time.



MSC M
p
q
r
e_1
e_2
e_3
e_4
$*$
e_1 $next$ $prev$ out_0 $+1$ out_1 $+3$
e_2 $next$ $prev$ out_0 $\perp$ out_1 $+1$
e_3 $next$ $prev$ out_0 $+1$ out_1 $\perp$
e_4 $next$ $prev$ out_0 $\perp$ out_1 $\perp$
$*$

$$p < q < r$$

$$Max(M) = ((4, poin(e_4)), (3, poin(e_3), (\emptyset, \perp))$$
$$Min(M) = ((1, poin(e_1)), (2, poin(e_2), (\emptyset, \perp))$$
$$length(M) = 4$$

Figure 4.1: *MSC* structure

3. $Max(M_{1 \cdot 2})$ is equal to $Max(M_2)$ on positions with nonempty values. On empty positions values from $Max(M_1)$ are used. In $\mathcal{O}(|\mathcal{P}|)$.

4. $Min(M_{1 \cdot 2})$ is equal to $Min(M_1)$ on positions with nonempty values. On empty positions values from $Min(M_2)$ are used. In $\mathcal{O}(|\mathcal{P}|)$.

5. $length(M_{1 \cdot 2}) = length(M_1) + length(M_2)$ (in $\mathcal{O}(1)$)

The sum of all operations is in $\mathcal{O}(|\mathcal{P}|)$, so a concatenation of a path of length n takes at most $\mathcal{O}(n|\mathcal{P}|)$ time.

4.2.2 Finding a deciding event in an *MSC*

Having an *MSC* M it needs to be checked whether it possesses a deciding event for some non–local choice q. If we look at the event set E as a partially ordered set $(E, <)$, we are trying to find a supremum for events in $Max(M)$ that are on processes from $First(q)$.

This can be achieved in time $\mathcal{O}(|E| \cdot |\mathcal{P}|)$. We allocate an array of size $length(M)$ that will represent the events in their topological order and with constant access to elements. A subset of processes is assigned to every event in the following way: We start assigning from the array end. Every event e has a $P(e)$ in its event set and gains all processes from events that are targets of its outgoing edges.

Because events are topologically ordered one pass through the events is enough. Moreover the first event that contains all processes from $First(q)$ is a deciding event for q.

4.3 Algorithm for decidable non–local choice

4.3.1 Highlighting deciding events

The answer whether a non–local choice node is decidable may not be enough. A designer of an *MSG* may need some further information. The Algorithm 3 depending on the decidability highlights various information about the *MSG*.

If the node is decidable, we highlight deciding events for every acyclic path from initial node to non–local choice node and a resolving event for every elementary cycle containing node q, to show the designer where he should plan to add deciding information into the communication.

If the non–local choice node is not decidable we highlight a path from initial node to non–local choice node without a deciding event or an elementary cycle containing non–local choice node that does not generate its own deciding events.

4.3.2 Total correctness

We need to show that Algorithm 3 terminates, but that is clear because we iterate only over sets of finite size. Partial correctness follows from results in Chapter 3.

4.3.3 Complexity

We will analyze the complexity of the algorithm. Given an *MSG* $G = (\mathcal{S}, \tau, s_0, s_f, L)$ over a common process set $\mathcal{P}$. We will denote the sum of all events of all *MSCs* in the *MSG* by e, c is the number of all elementary cycles containing non–local choice node q and p is the number of all acyclic paths from

Algorithm 3 Decidable non–local choice

```
 1: input non-local choice node $q$
 2: $c \leftarrow$ all acyclic paths
 3: for all $path \in c$ do
 4:    if $path$ contains a deciding event $e$ then
 5:       highlight $e$
 6:    else
 7:       highlight $path$
 8:       return $q$ is not decidable
 9: $c \leftarrow$ all elementary cycles
10: for all $cycle \in c$ do
11:    if $cycle$ generates deciding events $e$ then
12:       highlight $e$
13:    else
14:       highlight $cycle$
15:       return $q$ is not decidable
16: return $q$ is decidable
```

the initial node to node q. Both constants p and q are bounded by:

$$\sum_{i=1}^{n-1} \binom{n}{n-i+1}(n-i)!$$

Generating all acyclic paths takes $\mathcal{O}((|\mathcal{S}|+|\tau|)\cdot(p+1))$ time. Constructing a path and test whether it contains a deciding event takes $\mathcal{O}(|\mathcal{S}|\cdot|\mathcal{P}|+|\mathcal{P}|\cdot e)$. Lines 1 – 8 together are in $\mathcal{O}((|\mathcal{S}|+|\tau|)\cdot(p+1)+p(|\mathcal{S}|\cdot|\mathcal{P}|+|\mathcal{P}|\cdot e))$. After rearrangement we get $\mathcal{O}(p(|\mathcal{P}|(|\mathcal{S}|+e)+|\tau|))$.

Generating all distinct elementary cycles takes $\mathcal{O}((|\mathcal{S}|+|\tau|)\cdot(c+1))$ time. Deciding whether an elementary cycle generate its own deciding events can be done by running BFS [15] for every process. The complexity of BFS is in $\mathcal{O}(V+E)$, but because no vertex has out–degree and in–degree higher than 2, BFS runs on a path with e events in $\mathcal{O}(e)$. Together lines 9 – 16 run in $\mathcal{O}((|\mathcal{S}|+|\tau|)\cdot(c+1)+c(|\mathcal{S}|\cdot|\mathcal{P}|+|\mathcal{P}|\cdot e))$. That can be rearranged to $\mathcal{O}(c(|\mathcal{P}|(|\mathcal{S}|+e)+|\tau|))$.

The complexity of the algorithm is the sum of the two previous partial complexities:

$$\mathcal{O}((p+c)(|\mathcal{P}|(|\mathcal{S}|+e)+|\tau|)).$$

Chapter 5

Conclusion

We have presented a new approach to deal with non–local choice in *MSG* specifications. We assumed that we can add additional information into the communication that will help us to resolve the problem. We required the information to be bounded, to be able to easily implement such behavior.

Not all *MSGs* can be resolved in this way, therefore we introduced an algorithm that decides whether our approach can be applied to a specific non–local choice node. Moreover whenever our approach can be applied the algorithm advices the designer, where he should add information. When an *MSG* cannot be resolved we are able to print a witness, to help the designer easily solve the problem.

The complexity of the algorithm may be worse than exponential with respect to the number of nodes of the specification, but we believe that in common cases the algorithm will run acceptably fast.

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